

# Polarization Measurement Errors Due to Spatial and Temporal Misregistration

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## ABSTRACT

This paper focuses on potential false conclusions regarding polarization parameters obtained through the use of a digital camera. It outlines a methodology for calibrating, displaying and visualizing polarization profiles of natural, daylight scenes using a COTS (commercial-off-the-shelf) digital camera. It presents cause and effect scenarios related to spatial and temporal misregistration of polarization data and the acquisition of accurate polarization parameters, such as the degree of polarization and the polarization azimuth and ellipticity angles. Trade-offs between precise spatial registration and precise temporal registration are discussed.

## THE STOKES PARAMETERS

The human visual system can barely detect polarized light. Some observers can determine, through sighting Haidinger's brush<sup>1</sup>, that skylight is polarized. However, the human visual system is incapable of determining the complete polarization state of a light beam. To determine the state of polarization corresponding to points in a scene, three independent parameters must be determined for each point<sup>2</sup>. For example, the three independent parameters could be the amplitudes of the x and y components of the electric vector  $E_{0x}$  and  $E_{0y}$  and their phase difference,  $\delta$ , along the optical axis. A prominent method to determine these three independent parameters, using measurable quantities, is the Stokes method<sup>3</sup>. This method involves measuring four intensities of a light beam. Each measurement corresponds to the intensity of the beam after it passes through each of four different filter system arrangements. The four Stokes parameters, sometimes called  $S_0$ ,  $S_1$ ,  $S_2$  and  $S_3$ , are derived from these four measured intensities and form a four-element column vector in four-dimensional mathematical space. The Stokes parameters are applicable to a beam of light that is completely polarized, partially polarized or unpolarized; the beam may be monochromatic or polychromatic. The Stokes parameters for completely polarized light propagating along the +z-axis are

$$S_0 = E_{0x}^2 + E_{0y}^2 \quad S_1 = E_{0x}^2 - E_{0y}^2 \quad S_2 = 2 E_{0x} E_{0y} \cos \delta \quad S_3 = 2 E_{0x} E_{0y} \sin \delta \quad (1)$$

Normalized parameters are obtained by dividing  $S_0$ ,  $S_1$ ,  $S_2$  and  $S_3$  by  $S_0$ . For partially polarized light the degree of polarization  $P$  is given by<sup>4</sup>

$$P = \frac{I_{\text{Polarized}}}{I_{\text{Total}}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad (2)$$

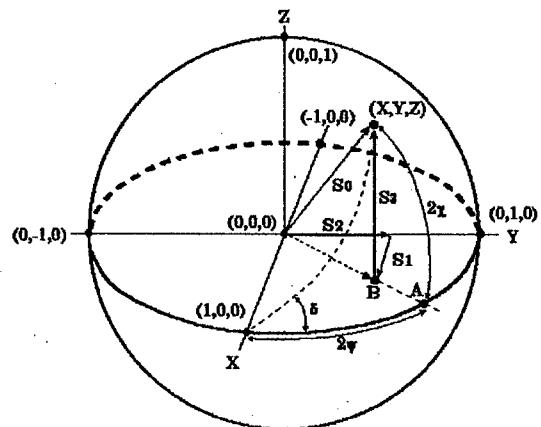


Figure 1 The relationship between the Stokes parameters and the Poincaré sphere.

where  $I_{\text{Polarized}}$  is the intensity of the polarized component and  $I_{\text{Total}}$  is the total intensity. When the Stokes parameters are normalized, the degree of polarization  $P$  becomes the radius of a sphere with center coincident to the typical Poincaré sphere (see Fig. 1) of unit radius ( $P = 1$ ). The inside of the Poincaré sphere can be used to represent the polarized portion of partially polarized light ( $P < 1$ ). The polarization forms on the surface of the inside spheres ( $P < 1$ ) are exactly the same as the polarization forms on the surface of the sphere of unit radius. From the geometry of Figure 1 and Eq. (2) it can be shown that for normalized Stokes parameters and for any degree of polarization

$$S_1 = P \cos 2\psi \cos 2\chi \quad S_2 = P \sin 2\psi \cos 2\chi \quad S_3 = P \sin 2\chi \quad (3)$$

In fact, Eq. (3) represents the spherical coordinates for any point on the surface or inside of the Poincaré sphere where  $(x, y, z) = (S_1, S_2, S_3)$ .

The polarization form associated with elliptical polarization is given by the following expression, as shown in Fig. 2

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y E_x}{E_{0y} E_{0x}}\right) \cos \delta = \sin^2 \delta \quad (4)$$

The polarization azimuth angle,  $\psi$ , and the ellipticity angle,  $\chi$ , are also defined in Fig. 2. Using Eq. (3),  $\psi$  and  $\chi$  can be determined from

$$\sin 2\chi = \frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}} \quad \text{and} \quad \tan 2\psi = \frac{S_2}{S_1} \quad (5)$$

The parameter  $\chi$  varies from  $+45^\circ$  to  $-45^\circ$ ; it is positive for right-handed polarization forms and negative for left-handed polarization forms. The parameter  $\psi$  varies from  $0^\circ$  to  $180^\circ$ ; it is  $0^\circ$  for horizontal polarization forms and  $90^\circ$  for vertical polarization forms.

## MEASURING THE STOKES PARAMETERS

The Stokes parameters can be measured using a linear polarizer and a retarder. If light is first transmitted through a linear retarder with its fast axis oriented at an angle  $\Omega$  with respect to the x-axis and then transmitted through a linear polarizer with its transmission axis oriented at an angle  $\theta$  with respect to the x-axis, the intensity of the transmitted light can be expressed as

$$I(\Omega, \theta, \epsilon) = E_{0x}^2 [\cos^2 \theta - \sin \Omega \cos \Omega \sin 2(\Omega - \theta) (1 - \cos \epsilon)] + E_{0y}^2 [\sin^2 \theta + \sin \Omega \cos \Omega \sin 2(\Omega - \theta) (1 - \cos \epsilon)] + E_{0x} E_{0y} [\cos 2(\Omega - \theta) \sin 2\Omega \cos \delta - \sin 2(\Omega - \theta) (\cos 2\Omega \cos \epsilon \cos \delta + \sin \epsilon \sin \delta)] \quad (6)$$

where  $\delta = (\phi_x - \phi_y)$  is the phase difference between the x and y components of the incident light and  $\epsilon$  is the phase difference produced by the retarder.  $I(\Omega, \theta, \epsilon)$  denotes an intensity measurement corresponding to a particular set of values for  $\Omega$ ,  $\theta$  and  $\epsilon$ . For a quartz retarder<sup>5</sup>

$$\epsilon = \frac{\pi}{2} \left( \frac{\lambda_T - 50.876}{\lambda - 50.876} \right) \quad (7)$$

where  $\lambda_T$  is the wavelength for  $\delta = 90^\circ$  (the tuned wavelength) and  $\lambda$  is the incident wavelength, both in nanometers. Figure 3 shows the phase difference for a quartz retarder as a function of the input wavelength for selected tuned wavelengths. The smallest variation in  $\epsilon$  occurs for a quartz retarder tuned to the blue end of the visible spectrum and for input wavelengths in the red end of the visible spectrum (the red channel of a COTS digital camera).

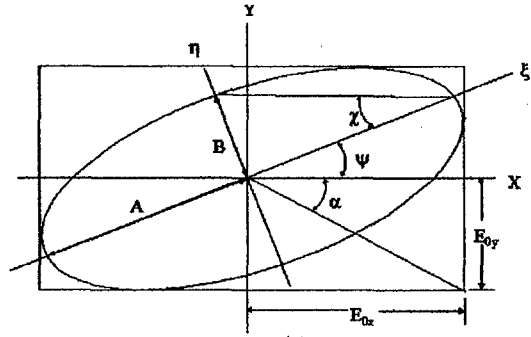


Figure 2 The polarization ellipse. The parameters  $\psi$ ,  $\chi$  are the polarization azimuth and ellipticity angles.

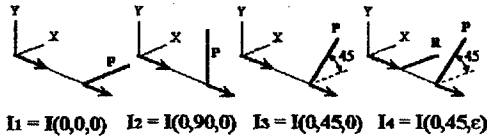
The Stokes parameters can be determined experimentally from the following four measurements, as shown in Fig. 4:

$$\begin{aligned} I_1 &= I(0, 0, 0) & I_2 &= I(0, 90, 0) \\ I_3 &= I(0, 45, 0) & I_4 &= I(0, 45, \varepsilon) \end{aligned} \quad (8)$$

In this arrangement, a retarder is inserted into the optical path for only the fourth measurement. The four measurements described in Eq. (8) are not the only four measurements that will lead to a determination of the Stokes parameter. In a previous work<sup>5</sup>, we describe a simple two-element variable filter system where neither the polarizer nor the retarder needs to be removed for any measurement.

Figure 5 shows an experimental setup to acquire this four-image sequence. It shows a digital camera, a linear polarizer, a hinged retarder and a hinged filter mounted on a tripod. The retarder and filter are hinged so that they can be moved quickly and precisely into and out of the optical path perpendicular to the optical axis. Substituting Eq. (8) into Eq. (6) and comparing the results with Eq. (1) confers that

$$\begin{aligned} S_0 &= I(0, 0, 0) + I(0, 90, 0) = I_1 + I_2 \\ S_1 &= I(0, 0, 0) - I(0, 90, 0) = I_1 - I_2; \\ S_2 &= 2 I(0, 45, 0) - S_0 = 2 I_3 - S_0 \\ S_3 &= \frac{2I_4 - S_0 - S_2 \cos \varepsilon}{\sin \varepsilon} \end{aligned} \quad (9)$$

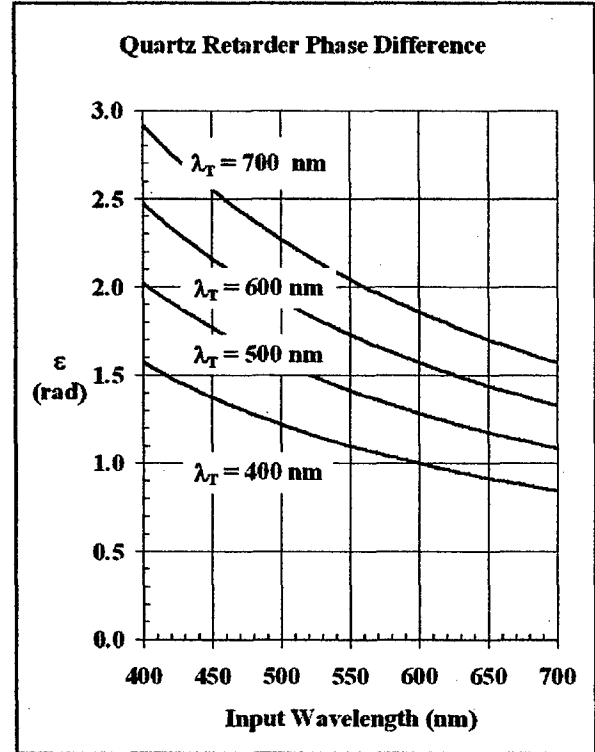


**Figure 4** The four-filter system used to determine the Stokes parameters. **P** is the transmission axis of a linear polarizer and **R** is the fast axis of a zero-order quartz quarter-wave retarder.

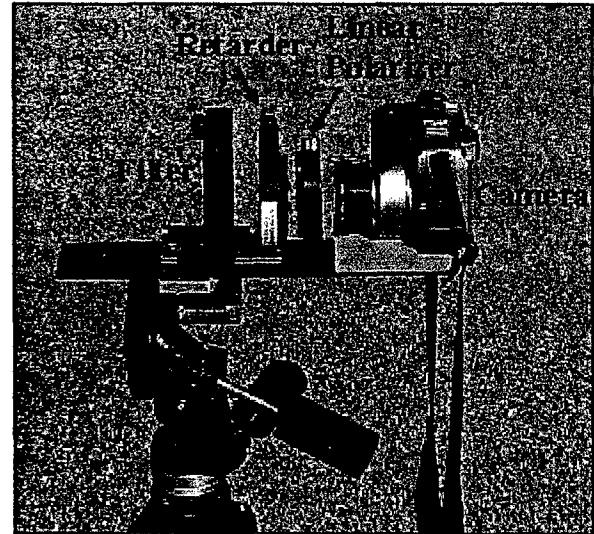
Since the Stokes parameters require quantities proportional to light intensity, it is essential that the digital camera be calibrated to convert RGB pixel values to intensities. This task is easily accomplished by photographing a white standard illuminated by a collimated white beam of light with a large number of different neutral density filters in front of the digital camera. The RGB to optical density conversion equation for an Epson 850Z digital camera was determined to be of the form

$$A x^2 + B xy + C y^2 + D x + E y + F = 0 \quad (10)$$

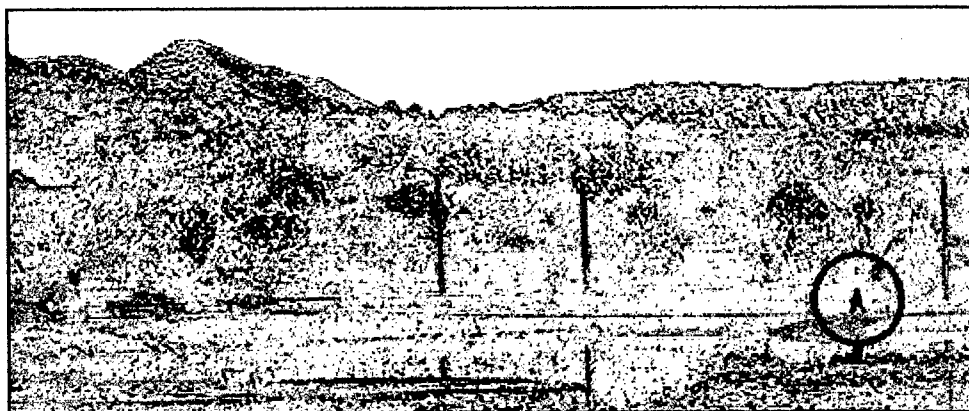
where  $x$  and  $y$  correspond to optical densities and RGB values respectfully. The coefficients in Eq. (10) are different for each color channel of the digital camera. Using the fixed white balance option of the Epson 850Z, the coefficients obtained for the green channel are:



**Figure 3.** Phase difference produced by a quartz retarder tuned to selected wavelengths.



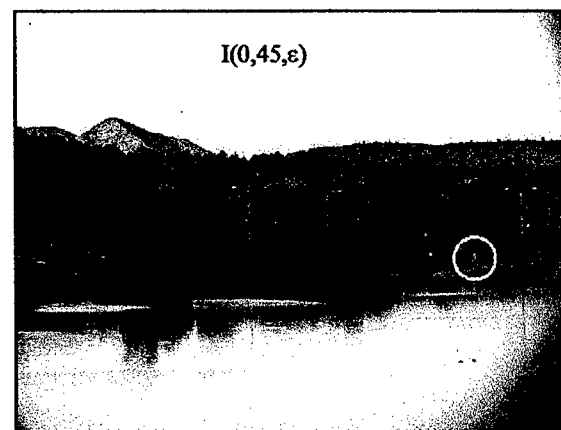
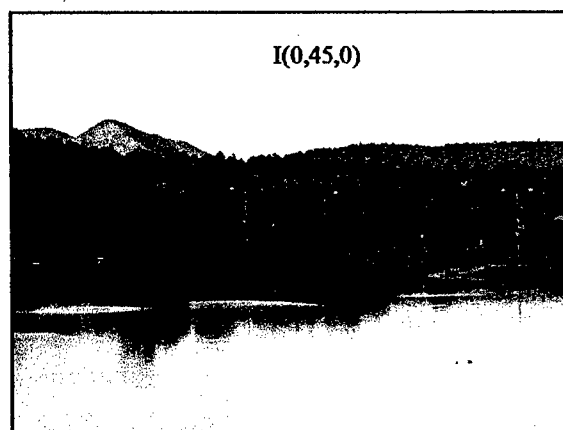
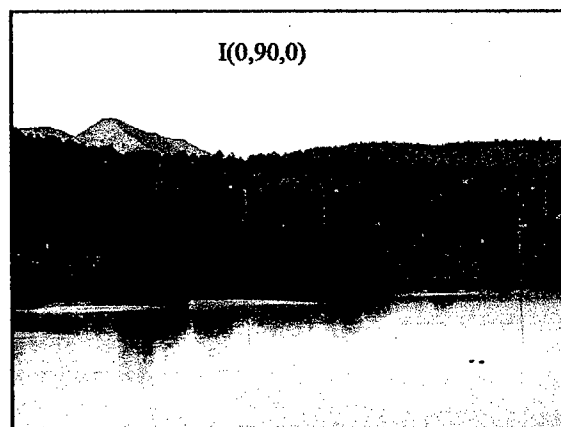
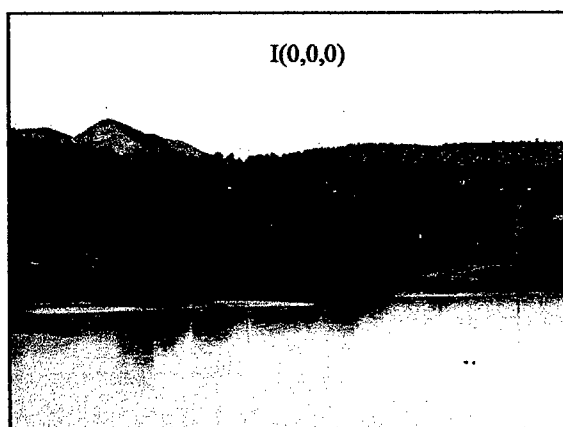
**Figure 5** An experimental camera setup to acquire the four-image sequence for obtaining the Stokes parameters associated with each pixel in a scene.



The polarization ellipticity angle



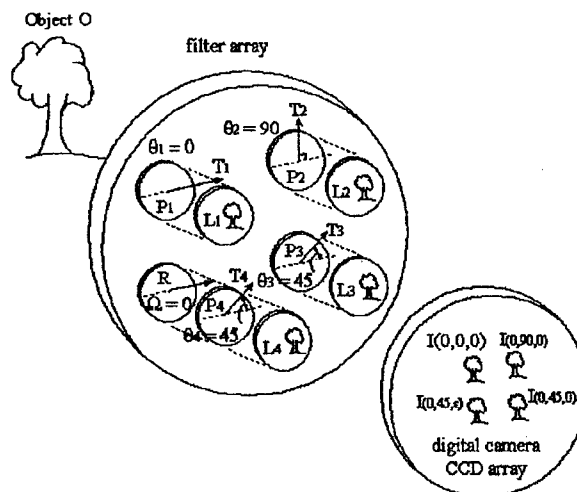
Color map for the polarization ellipticity angle



**Figure 14** The four Stokes-filter images of Green Valley Park at 6:16 AM on 09-16-02 and the resulting polarization ellipticity angle for the scene. These four images were acquired at 10-second intervals. The person in the  $I(0,45,e)$  image does not appear in the other images.

## CONCLUSION

This paper has outlined a methodology for measuring the four Stokes parameters and calibrating, displaying and visualizing polarization profiles of natural, daylight scenes using a COTS digital camera. Measurement errors due to spatial and temporal misregistration of polarization data have been described and examples of each have been given. It has been shown that when the intensity increases between image-acquisition, the  $\chi$ -values increase and produce pseudo right-handed elliptically polarized light. When the intensity decreases between image-acquisition, the  $\chi$ -values decrease and produce pseudo left-handed elliptically polarized light. Trade-offs between precise spatial registration and precise temporal registration have also been discussed. Current studies strive to reduce the time required to obtain the four Stokes parameters and to minimize spatial and temporal misregistration.



**Figure 15.** An apparatus that simultaneously creates four different attenuated images of a scene. Corresponding pixel-values from each image are used to calculate the Stokes parameters for each pixel location.

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